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The computer simulation of drill column dragging in inclined bore-holes with geometrical imperfections

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A B S T R A C T

The problem about identification of resistance forces acting on a drill column moving in an inclined bore-hole is stated. It is supposed that the well trajectories can have geometrical imperfections in the shape of cylindrical spiral or plane cosinusoidal curves. The system of ordinary differential equations is derived on the basis of the theory of curvilinear flexible elastic rods. It permits one to describe static effects of the drill column bending accompanying the processes of its raising, lowering and rotating inside the bore-hole. Through the use of this system the direct and inverse problems of the drill column deforming are formulated for calculation of internal and external resistance forces acting on the drill column tube. The methods for numerical solution of the constructed equations are elaborated. With their use the phenomena of the drill columns motion and their frictional seizure inside the bore-holes are simulated for different geometrical imperfections and relations between the velocities and directions of their rotation and axial motion.

1. Introduction

In the last century the time of easy oil and natural gas terminated (Chow et al., 2003; Kerr, 2005). Inasmuch as the reserves of hydrocarbon fuels in easy-to-extract basins approach to depletion the deposits located at the depths of 10 km become to be promising. Now the old bore-hole drilling using vertical wells is being redeveloped by horizontal and inclined wells (McDermott et al., 2005). But the experience gained while drilling vertical wells is not useful for drilling horizontal ones, because mechanical behavior of a drill column (DC) with curvilinear axial line acquires a series of specificities leading to critical situations. So the failure rate at the curvilinear bore-hole driving achieves 1 in 3 holes (Mohiuddin et al., 2007). Taking into consideration that lengths of the modern curvilinear bore-holes are planned to approach 15 km and their costs exceed $60 million, it can be concluded that the problem of computer simulation of the drill column behavior is very urgent. At the same time the efforts to solve it present considerable difficulties determined by a number of factors (Iyoho et al., 2005). Among them the large length of the DC is the main one. As the present day drill columns can be compared by conditions of geometrical similarity with a human hair, usually the computer simulations of internal and external forces acting on them are performed with the use of simplified mathematical models based on the theory of absolutely flexible threads (Bernt and Anderson, 2001; Sheppard et al., 1987). Analysis of these forces is performed only on the basis of investigations of geometrical peculiarities of the bore-hole axis line without considering the contribution of elastic forces generated during raising-lowering operations and the DC rotation. As this takes place, the designs of bore-holes with simple outlines of catenary, brachistochrone, clothoid and the Cornu spiral are performed (Bernt and Anderson, 2001; Sheppard et al., 1987; Choe et al., 2005; Sawaryn and Thorogood, 2005). In papers (Stuart et al., 2003; Prassl et al., 2005; Brett et al., 1989) more general approach is used which is based on the consideration that the well axis outline represents a smooth shape combined from segments of straight lines and circular or catinary curves. It is referred to as a minimum curvature method. With its use explicit analytical equations are derived to model drill column (thread) tension and friction forces for hoisting or lowering operations. In addition, explicit expressions for drag and torque are developed for combined axial motion and rotation of the drill column. Using these equalities, the total drag and torque are derived from the sum of their separate contributions from each section of the hole. Different algorithms and software programs are presented. Several examples demonstrate the use of the analytical models. It is shown that any change of direction in the well path contributes to increased friction.

The formulated conclusion underlines weakness of the used approach which is based on assumption of well trajectory smoothness and possibility to neglect bending stiffness of the drill column tube. In actual practice, the axial trajectories of bore-holes do not represent lines with smooth geometry because geometrical

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imperfections are inserted into their outlines in drilling. They can be caused by distortions of the bit geometry, dynamic imbalance of the bottom-hole-assembly and physical non-homogeneities of the rock medium drilled. Usually they have the modes of local spiral and cosinusoidal curvings imposed on the designed trajectory of the bore-hole. Depending on their amplitudes and steps, they can lead to essential local bending of the DC and emergence of additional contact and friction forces which can be detected and described only with the use of the elastic curvilinear rod theory.

The second important reason for application of this theory is the possibility of the friction force regulation via the simultaneous axial motion and rotation of the DC during its lowering and raising (Stuart et al., 2003; Prassl et al., 2005; Brett et al., 1989). These procedures are more precisely simulated only on the basis of elastic rod models. But their application is associated with the necessity to use non-linear high order differential equations and, as shown below, to formulate inverse problem for part of the required variables. We were not able to find discussions of similar questions in scientific literature. Analogous problems are considered in papers (Mitchell, 2004, 2006, 2008a,b), where post-critical states followed by spiral bending of immovable DCs inside cylindrical cavities are investigated.

At oil and gas extraction from hyper-deep levels, the efficiency enhancement of bore-holes drilling is associated with solution of the problems on revealing the critical regimes of the drill column functioning and elaboration of measures for their prevention. This paper considers the model problem of quasi-static equilibrium and deforming of elongated (down to 10 km) drill columns in inclined bore-holes studied with allowance made for effects of non-uniform gravity and friction forces and action of a torque. The investigations are based on the statement of direct problems for one part of the variables and inverse problems for the others. Special attention is paid to the question of analysis of friction force influence on mechanical behaviour of the drill columns during their raising, lowering and rotating in inclined bore-holes with spiral and cosinusoidal geometrical imperfections. As this takes place, the questions of estimation of internal and external forces and moments acting on the DC are of chief interest.

Simulation of the resistance forces and quasi-static phenomena attending the bore-hole drilling allows one to solve the fundamental problems like provision of the required geometry for the bore-hole axis, decrease of the contact and friction interaction forces between the DC and bore-hole wall and avoiding the drill column tube seizure inside the bore-hole. Owing to these effects, the DC tube wearing is reduced, undesirable curving the axial line of the bore-hole is excluded and, as a consequence, the heavy emergencies are prevented during the drilling processes.

2. Statement of direct and inverse problems about elastic drill column dragging in an inclined bore-hole

To set up the problem about calculation of the external and internal quasistatic forces acting on a curvilinear drill column at different stages of its functioning, take into account that usually the curvature radii of the bore-hole axes exceed hundreds of metres, the DC lengths equal several kilometres, but the clearances between the DC and bore-hole surfaces generally are 0.1–0.15 m. This permits one to assume that the bore-hole trajectory is prescribed and in the state of the DC operation its elastic line acquires the shape of the bore-hole axis line. Then the functions of the internal moments and shear forces acting in the DC are calculated via simple formulas and can be considered to be known. In this case the direct problem of the theory of quasi-statics of curvilinear flexible rods can be stated for determination of internal longitudinal force and torque, while the external forces of contact and friction interaction between the DC and bore-hole wall can be calculated through the statement of an inverse problem. With this approach the behavior of the DC in curvilinear bore-hole can be described, the zones of possible seizure of the DC can be detected and the measures for the DC jarring and release can be designed. In this case it is conveniently to use the theory of curvilinear flexible rods to describe the stress–strain state of the DC. The foundations of this theory are presented in (Gulyayev et al., 1992).

To describe the mechanics of the DC, it is suitable to use jointly external and internal geometries, applying the first one to individualize the points of the curvilinear tubular rod axis and the second one to describe its geometry in the deformed state.

The internal geometry of the rod is specified by the co-ordinate s, measured as the length of the axial line from the initial to the current point, and a moving right-handed co-ordinate system (u, v, w), whose orientation is rigidly connected with the examined cross-section at every point of the tube axial line. The origin of this system lies at the center of gravity of the cross-section area, the u- and v- axis are directed along the principal central axes of inertia of the cross-section area, and the w - axis is directed along the tangent to the elastic line. In this case the co-ordinate s is a concomitant one. The external geometry of the rod determines the location of each of its points and the entire elastic line in the fixed inertial co-ordinate system $Oxyz$.

The Frenet natural trihedron of the elastic line of the rod with unit vectors of the principal normal $\boldsymbol{n}$, binormal $\boldsymbol{b}$ and tangent $\boldsymbol{\tau}$ is also introduced.

If the axial line of the rod (Fig. 1) is preset by the equalities

$$x = x(s), \quad y = y(s), \quad z = z(s), \tag{1}$$

its geometrical characteristics can be determined via the formulae

$$\frac{1}{R} = \sqrt{(x'')^2 + (y'')^2 + (z'')^2}, \quad \frac{1}{T} = R^2 \begin{vmatrix} x' & y' & z' \\ x'' & y'' & z'' \\ x''' & y''' & z''' \end{vmatrix}$$

$$p = \frac{1}{R}\sin\chi, \quad q = \frac{1}{R}\cos\chi, \quad r = \frac{1}{T} + \frac{d\chi}{ds}. \tag{2}$$

Here χ is the angle between the $\boldsymbol{n}$ unit vector and the u-axis; R is the curvature radius; T is the torsion radius; p, q, r are the curvatures and torsion of the bore-hole axis line; the superindex prime denotes differentiation with respect to s.

Assume that the spatial outline of the axial line of the made hole is known and it is defined by the radius-vector $\boldsymbol{\rho} = \boldsymbol{\rho}(s)$ or scalar Eq. (1). Then it is possible to calculate the functions $R(s)$, $T(s)$ with the use of formulas (2) and to determine the vectors

$$\boldsymbol{\tau} = d\boldsymbol{\rho}/ds, \quad \boldsymbol{n} = Rd^2\boldsymbol{\rho}/ds^2, \quad \boldsymbol{b} = \boldsymbol{\tau} \times \boldsymbol{n}. \tag{3}$$

It is also useful to remember, that Eq. (3) are not independent, inasmuch as they have six first integrals

$$|\boldsymbol{\tau}| = 1, \quad |\boldsymbol{n}| = 1, \quad \boldsymbol{\tau} \cdot \boldsymbol{n} = 0, \quad \boldsymbol{\tau} \times \boldsymbol{n} = \boldsymbol{b} \tag{4}$$

issuing from the condition of the Frenet basis orthonormality.

At statement of the problem about static deforming the DC tube inside the bore-hole channel, it is assumed that the tube cross-section dimensions are very small in comparison with its length and curvature radius of its axial line. Because of this under action of

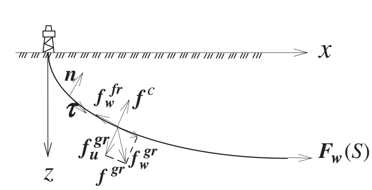

Fig. 1. Geometrical scheme of a drill column in an inclined bore-hole.

external forces the DC tube acquires the shape differing significantly from its initial one. This allows to consider that the tube material strains are elastic and to neglect elongations of the tube axis line. As a result the longitudinal force in the tube is determined from an equilibrium equation, thereupon (if necessary) appropriate axial strains can be found and axial elongation be calculated.

The equations of bending an elastic tubular rod with distributed forces $\boldsymbol{f}$ and moments $\boldsymbol{m}$ are written in the form of the system of equilibrium equations (Gulyayev et al., 1992)

$$d\boldsymbol{F}/ds = -\boldsymbol{f}, \quad d\boldsymbol{M}/ds = -\boldsymbol{\tau} \times \boldsymbol{F} - \boldsymbol{m}, \tag{5}$$

equations of elasticity

$$\begin{aligned} &M_u = Ap, \quad M_v = Bq, \quad M_w = Cr, \\ &A = EI_u, \quad B = EI_v, \quad C = GI_w \end{aligned} \tag{6}$$

and equations of kinematics

$$\begin{aligned} &d\boldsymbol{\tau}/ds = \boldsymbol{n}/R, \quad d\boldsymbol{n}/ds = -\boldsymbol{\tau}/R + \boldsymbol{b}/\boldsymbol{T}, \\ &d\boldsymbol{b}/ds = -\boldsymbol{n}/T, \quad d\boldsymbol{\rho}/ds = \boldsymbol{\tau}, \end{aligned} \tag{7}$$

where $\boldsymbol{F}$, $\boldsymbol{M}$ are the vectors of the internal forces and moments with components F_u, F_v, F_w and M_u, M_v, M_w, respectively; A, B, C are the parameters of the flexural and torsional stiffnesses; E is the elasticity module of the material; G is the shear module; I_u, I_v are the inertia moments of the rod cross-section; I_w is the polar inertia moment. In our case $A = B$ for the cross-section of the DC tube.

In use of Eq. (5) it is necessary to take into account that they are written out in the (u, v, w) coordinate system, which changes from a point to point, so the total derivatives $d\boldsymbol{F}/ds$ and $d\boldsymbol{M}/ds$ should be calculated through the use of the equalities

$$d\boldsymbol{F}/ds = \tilde{d}\boldsymbol{F}/ds + \boldsymbol{\omega_\chi} \times \boldsymbol{F}, \quad d\boldsymbol{M}/ds = \tilde{d}\boldsymbol{M}/ds + \boldsymbol{\omega_\chi} \times \boldsymbol{M},$$

which stem from the Euler equalities known in classical mechanics. Here $d\widetilde{\boldsymbol{F}}/ds$ and $d\widetilde{\boldsymbol{M}}/ds$ are the local derivatives; $\boldsymbol{\omega_\chi}$ is the Darboux vector which equals

$$\boldsymbol{\omega_\chi} = \boldsymbol{b}/R + (1/T + d\chi/ds)\boldsymbol{\tau}. \tag{8}$$

So the vectors $\boldsymbol{F}$, $\boldsymbol{M}$, $d\widetilde{\boldsymbol{F}}/ds, d\widetilde{\boldsymbol{M}}/ds$ and $\boldsymbol{\omega}_\chi$ have the components F_u, F_v, F_w, M_u, M_v, M_w, dF_u/ds, dF_v/ds, dF_w/ds, dM_u/ds, dM_v/ds, dM_w/ds and p, q, r, correspondingly. Then Eq. (5) can be represented in the scalar form

$$\begin{aligned} &dF_u/ds = -qF_w + rF_v - f_u, \\ &dF_v/ds = -rF_u + pF_w - f_v, \\ &dF_w/ds = -pF_v + qF_u - f_w \end{aligned} \tag{9}$$

for the force equilibrium and in the same form

$$\begin{aligned} &dp/ds = (Brq - Cqr + F_v - m_u)/A, \\ &dq/ds = (Cpr - Arp - F_u - m_v)/B, \\ &dr/ds = (Aqp - Bpq - m_w)/C \end{aligned} \tag{10}$$

for the moments equilibrium.

Eq. (7) are also brought to the scalar form

$$\begin{aligned} &d\tau_x/ds = n_x/R, \quad d\tau_y/ds = n_y/R, \quad d\tau_z/ds = n_z/R, \\ &dn_x/ds = -\tau_x/R + b_x/T, \quad dn_y/ds = -\tau_y/R + b_y/T, \\ &dn_z/ds = -\tau_z/R + b_z/T, \quad db_x/ds = -n_x/T, \\ &db_y/ds = -n_y/T, \quad db_z/ds = -n_z/T, \\ &dx/ds = \tau_x, \quad dy/ds = \tau_y, \quad dz/ds = \tau_z. \end{aligned} \tag{11}$$

The system of constitutive Eqs. (9)–(11) describes nonlinear bending of a rod under action of external distributed forces f_u, f_v, f_w and moments m_u, m_v, m_w. Its total order is eighteen, though it is intended for determination of twelve required variables (three internal forces F_u, F_v, F_w; three internal moments M_u, M_v, M_w; three coordinates x, y, z and three kinematic functions, which are usually selected according to the investigater's taste). Therefore, the set of equations and unknowns form a redundant set.

Six first integrals (4) permit to reduce the system order to twelve, but in this case it will gain very cumbersome form. Because of this, in practice (Gulyayev et al., 1992) usually the total system of Eqs. (9)–(11) is used and first integrals (4) are applied to check the solution accuracy.

If the external distributed forces f_u, f_v, f_w and moments m_u, m_v, m_w acting on the elastic rod are prescribed and it is necessary to determine its internal forces, moments and geometry functions, the boundary value problem should be stated for system (9)–(11). In this case Eqs. (9)–(11) are integrated in the limits of the rod length and this problem is called the direct one. But if the external forces and moments are not known and some of the geometric parameters of the rod axis are predetermined then kinematic differential Eq. (11) are converted to identities and Eqs. (9) and (10) are used for calculation of external (contact) forces and moments. In the theory of differential equations and classical mechanics such problems are called inverse ones.

To formulate the inverse problem for the DC inserted into the bore-hole, combine the forces f_u, f_v, f_w from the known gravity forces f_u^{gr}, f_v^{gr}, f_w^{gr}, unknown forces f_u^c, f_v^c of the tube contact with the bore-hole wall and friction force f_w^{fr} (Fig. 1). They can be introduced in the form

$$f_u = f_u^{gr} + f_u^c, \quad f_v = f_v^{gr} + f_v^c, \quad f_w = f_w^{gr} + f_w^{fr}. \tag{12}$$

The components f_u^{gr}, f_v^{gr}, f_w^{gr} are calculated through projecting the gravity intensity vector $\boldsymbol{f}^{gr} = \gamma g\boldsymbol{k}$ on the axes u, v, w

$$f_u^{gr} = \gamma g n_z, \quad f_v^{gr} = \gamma g b_z, \quad f_w^{gr} = \gamma g \tau_z, \tag{13}$$

where $\gamma = \gamma_t - \gamma_l$; γ_t is the tube mass per unit length; γ_l is the mud (washing liquid) mass per the tube unit length; g = 9.81 m/s^2 is the acceleration of gravity.

Upon taking into account correlations (2), (12), Eq. (9) are converted to the form

$$\left.\begin{aligned} &\tfrac{dF_u}{ds} = \left(\tfrac{1}{T} + \tfrac{d\chi}{ds}\right)F_v - \tfrac{1}{R}\cos\chi \cdot F_w - f_u^{gr} - f_u^c, \\ &\tfrac{dF_v}{ds} = \tfrac{1}{R}\sin\chi \cdot F_w - \left(\tfrac{1}{T} + \tfrac{d\chi}{ds}\right)F_u - f_v^{gr} - f_v^c, \\ &\tfrac{dF_w}{ds} = \tfrac{1}{R}\cos\chi \cdot F_u - \tfrac{1}{R}\sin\chi \cdot F_v - f_w^{gr} - f_w^{fr}. \end{aligned}\right\} \tag{14}$$

In the theory of elastic beams, it is usually assumed that m_u=0, m_v=0. Then Eq. (10) are transformed to the system

$$\left.\begin{aligned} &\tfrac{d}{ds}\left(\tfrac{\sin\chi}{R}\right) = \tfrac{A-C}{A}\tfrac{\cos\chi}{R}\left(\tfrac{1}{T} + \tfrac{d\chi}{ds}\right) + \tfrac{F_v}{A}, \\ &\tfrac{d}{ds}\left(\tfrac{\cos\chi}{R}\right) = \tfrac{C-A}{A}\tfrac{\sin\chi}{R}\left(\tfrac{1}{T} + \tfrac{d\chi}{ds}\right) - \tfrac{F_u}{A}, \\ &\tfrac{d}{ds}\left(\tfrac{1}{T} + \tfrac{d\chi}{ds}\right) = -\tfrac{m_{fr}}{C}. \end{aligned}\right\} \tag{15}$$

The system of six Eqs. (14), (15) contains eight unknown variables F_u, F_v, F_w, χ, f_u^c, f_v^c, f_w^{fr}, m_{fr}. So it should be supplemented by two equalities determining the distributed friction force f_w^{fr} and moment m_{fr}. To calculate them, it is necessary to determine the total friction force module

$$|f^{fr}| = |\mu f^c| = \mu\sqrt{\left(f_u^c\right)^2 + \left(f_v^c\right)^2}, \tag{16}$$

then to decompose it to the axial $\left(f_w^{fr}\right)$ and circumferential $\left(f_{cir}^{fr}\right)$ components according to the Coulombic friction law

$$|f_w^{fr}| = \mu\sqrt{\left(f_u^c\right)^2 + \left(f_v^c\right)^2} \cdot \left|\dot{w}/\sqrt{\dot{w}^2 + (\omega \cdot d/2)^2}\right|, \tag{17}$$

$$\left|f_{cir}^{fr}\right| = \mu\sqrt{\left(f_u^c\right)^2 + \left(f_v^c\right)^2} \cdot \left|\omega d/2\sqrt{\dot{w}^2 + (\omega \cdot d/2)^2}\right|. \tag{18}$$

In Eqs. (16)–(18) μ is the dry friction coefficient; $\dot{w}$ - the velocity of longitudinal motion of the DC inside the bore-hole; ω - the angular velocity of the DC rotation; d - the external diameter of the DC tube.

The friction moment m_{fr} is expressed through the force $\left|f_{cir}^{fr}\right|$

$$|m_{fr}| = \left|f_{cir}^{fr}\right| \cdot d/2 = \mu\sqrt{\left(f_u^c\right)^2 + \left(f_v^c\right)^2} \cdot \left|\omega d^2/4\sqrt{\dot{w}^2 + (\omega \cdot d/2)^2}\right|$$

Then two equalities are added to system (14) and (15)

$$f_w^{fr} = \pm\mu\left[\sqrt{\left(f_u^c\right)^2 + \left(f_v^c\right)^2} \cdot \dot{w}/\sqrt{\dot{w}^2 + (\omega \cdot d/2)^2}\right],$$
$$m_{fr} = \pm\mu\left[\sqrt{\left(f_u^c\right)^2 + \left(f_v^c\right)^2} \cdot d^2 \cdot \omega/4\sqrt{\dot{w}^2 + (\omega \cdot d/2)^2}\right], \tag{19}$$

where signs "+" and "−" are chosen according to the directions of the DC rotation and axial movement at the regimes of its raising and lowering. In computer simulations it was considered that the analyzed regimes were steady and as a consequence axial and rotational velocities of the DC elements did not vary along their axial lines.

Equalities (19) testify that the friction force and moment can be regulated through the appropriate selection of the ratio $v = \dot{w}/(\omega d/2)$ between the velocities of axial movement of the DC and its rotation.

System (14), (15), (19) with appropriate boundary conditions allows one to set direct and inverse problems about calculation of internal and external forces, acting on the DC with different regimes of ratios between the longitudinal motion velocity $\dot{w}$ and rotation velocity ω during its lowering or raising. Below it is used for investigation of the bore-hole geometry imperfection influence on the resistance forces impeding the DC motion. The spiral and cosinusoidal imperfections of the bore-hole axis line are considered.

3. Raising and lowering a drill column in a bore-hole with spiral imperfections

At design of drilling the directional and horizontal bore-holes, the condition of minimization of friction forces resisting moving and rotating a DC is one of the most important requirements. As it is shown below, these forces depend essentially on the boundary conditions at the points of the DC suspension and its contact interaction with the bore-hole bottom, along with the short wave geometrical imperfections of the DC axis line.

Assume that the bore-hole segment is conventionally rectilinear and is supposed to have imperfections in the shape of a spiral curve of diameter $2a$ and pitch λ in the coordinate system $Oxyz$ (Fig. 2). The spiral reference axis is inclined at the angle β to the horizontal axis Ox. At computer simulation the angles α, β, diameter $2a$ and pitch $\lambda = 2\pi a\, tg\alpha$ can be varied with the aim to investigate their values influence on the DC stress–strain state and the resistance forces emerging at its raising and lowering.

The equalities of the bore-hole line are chosen as

$$x = s \cdot \sin\alpha \cdot \cos\beta + \left[a\sin\left(\frac{\cos\alpha}{a} \cdot s\right)\right] \cdot \sin\beta, \quad y = a \cdot \cos\left(\frac{\cos\alpha}{a} \cdot s\right),$$
$$z = -s \cdot \sin\alpha \cdot \sin\beta + \left[a\sin\left(\frac{\cos\alpha}{a} \cdot s\right)\right] \cdot \cos\beta. \tag{20}$$

This line is characterized by constant radii

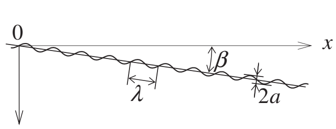

Fig. 2. The segment of inclined bore-hole with spiral imperfections.

$$R = \frac{a}{\cos^2\alpha}, \quad T = \frac{a}{\sin\alpha \cdot \cos\alpha}. \tag{21}$$

Then system (15) of moment equilibrium equations can be reduced to the form

$$\frac{1}{R}\cos\chi\frac{d\chi}{ds} = \frac{(A-C)}{A}\frac{\cos\chi}{R}\left(\frac{1}{T} + \frac{d\chi}{ds}\right) + \frac{F_v}{A},$$
$$-\frac{1}{R}\sin\chi\frac{d\chi}{ds} = \frac{(C-A)}{A}\frac{\sin\chi}{R}\left(\frac{1}{T} + \frac{d\chi}{ds}\right) - \frac{F_u}{A}, \tag{22}$$
$$\frac{d^2\chi}{ds^2} = -\frac{m_{fr}}{C}.$$

Upon introducing designations $h_1 = \chi$, $h_2 = d\chi/ds$ and performing substitutions

$$p = \frac{\cos^2\alpha}{a}\sin\chi, \quad q = \frac{\cos^2\alpha}{a}\cos\chi, \quad r = \frac{\sin\alpha \cdot \cos\alpha}{a} + \frac{d\chi}{ds},$$

the system of Eqs. (10), (13)–(15) is transformed to the constitutive equations

$$\frac{dh_1}{ds} = h_2,$$
$$\frac{dh_2}{ds} = -\frac{m_{fr}}{C}, \tag{23}$$
$$\frac{dF_w}{ds} = \frac{1}{R}\cos h_1 F_u - \frac{1}{R}\sin h_1 F_v - f_w^{fr} - f_w^{gr},$$

where

$$F_u = \frac{A}{R}\sin h_1 \cdot h_2 + \frac{(C-A)}{R}\sin h_1\left(\frac{1}{T} + h_2\right),$$
$$F_v = \frac{A}{R}\cos h_1 \cdot h_2 + \frac{(A-C)}{R}\cos h_1\left(\frac{1}{T} + h_2\right),$$
$$f_u^c = \frac{\sin h_1}{R}m_{fr} + \frac{C}{RT}\cos h_1 \cdot h_2 - \frac{(A-C)}{RT^2}\cos h_1 - \frac{1}{R}\cos h_1 \cdot F_w - f_u^{gr}, \tag{24}$$
$$f_v^c = \frac{1}{R}\sin h_1 \cdot F_w + \frac{\cos h_1}{R}m_{fr} - \frac{C}{RT}\sin h_1 \cdot h_2 + \frac{(A-C)}{RT^2}\sin h_1 - f_v^{gr},$$
$$f_w^{fr} = \pm\mu\left[\sqrt{\left(f_u^c\right)^2 + \left(f_v^c\right)^2} \cdot \dot{w}/\sqrt{\dot{w}^2 + (\omega \cdot d/2)^2}\right],$$
$$m_{fr} = \pm\mu\left[\sqrt{\left(f_u^c\right)^2 + \left(f_v^c\right)^2} \cdot d^2 \cdot \omega/4\sqrt{\dot{w}^2 + (\omega \cdot d/2)^2}\right].$$

Owing to the statement of inverse problem (24) for some variables describing equilibrium of a drill column in a curvilinear bore-hole with the predetermined geometry, it became possible to reduce the sixth order differential equation system (9), (10) to system (23) of the third order with prescribed boundary conditions for the functions h_1, h_2, F_w at the ends $s=0$ or $s = S$. The problem is simplified, if the end $s = S$ is free during performing the lowering–raising operations. Then $F_w(S) = 0$, $M_w(S) = 0$ and it is necessary to find the force $F_w(0)$ and the moment $M_w(0)$, which make it possible to lower or to raise the DC. Then the equalities

$$h_1(S) = 0, \quad h_2(S) = 0, \quad F_w(S) = 0, \quad F_u(S) = F_v(S) = 0,$$
$$f_u^c(S) = -f_u^{gr}(S), \quad f_v^c(S) = -f_v^{gr}(S),$$
$$f_w^{fr}(S) = \pm\mu\sqrt{\left[f_u^{gr}(S)\right]^2 + \left[f_v^{gr}(S)\right]^2} \cdot \dot{w}/\sqrt{\dot{w}^2 + (\omega d/2)^2}, \tag{25}$$
$$m^{fr}(S) = \pm\mu\sqrt{\left[f_u^{gr}(S)\right]^2 + \left[f_v^{gr}(S)\right]^2} \cdot \omega d^2 \Big/ \left[4\sqrt{\dot{w}^2 + (\omega d^2/2)^2}\right]$$

can be used as initial conditions. They permit one to formulate the Cauchy problem for differential Eq. (23) with equalities (24) and initial conditions (25).

The stated problem was solved by the Runge–Kutta method. The integration step was chosen by the trial-and-error method on condition of the calculation convergence. It equaled $\Delta s = S/$

Table 1
The results of calculation of the lowering–raising operations in the spiral bore-hole of the length S = 10,000 m (a = 5 m, λ = 100 m, v = 1/75).

Operation	β (deg)	ω (sign)	$F_w(0)$ (kN)	$M_w(0)$ (Nm)	χ (rad)	$\lvert f^c\rvert_{max}$ (N/m)	$\left\lvert f_w^{fr}\right\rvert_{max}$ (N/m)
Lowering	0	+	−9.284	−53,299	−679	415.4	1.11
		_	−9.070	51,949	−468	311.9	1.36
	5	+	184.450	−361,858	−1086	4189.4	11.17
		_	190.460	323,957	−99	3599.2	9.60
	11.3	+	422.660	−776,771	−1645	8863.1	23.63
		_	435.300	697,041	417	7590.1	20.21
	25	+	918.380	−1,641,889	−2813	18577.0	49.53
		_	944.840	1,474,941	1495	15881.0	42.34
Raising	0	+	9.633	−66,094	−694	765.8	2.04
		_	8.515	59,042	−458	618.5	1.65
	5	+	322.750	−510,481	−1236	6790.7	18.11
		_	312.830	447,942	31	5645.2	15.05
	11.3	+	718.020	−1,086,713	−1951	14409.1	38.41
		_	697.410	956,700	682	11977.2	31.93
	25	+	1541.300	−2,288,438	−3445	30268.0	80.71
		_	1498.400	2,017,662	2044	25150.0	67.06

8000. The calculation results were used for analysis of the resistance forces depending on the ratio between the velocities of the axial movement $\dot{w}$ and rotation ω.

There is a great variety of determining factors used in drilling designs. They differ essentially by the bore-hole diameters (up to 40 cm), DC materials (steel, aluminium, titanium, composite), horizontal distances from the rig tower (exceeding 12 km), friction coefficients (μ = 0.2 − 0.25), angles of the bore-hole axis inclination ($0 \leqslant \beta \leqslant 90^\circ$) and others.

In our model problem the typical determining factors were chosen for the investigation: S=10,000 m; g=9.81 m/s^2; a=5 m; λ=100 m; d=0.1683 m; A=3.284·10^{6}Pa m^4; C=2.526·10^6 Pa m^4; γ = 29.8 kg/m; μ = 0.2. The gravity force of the whole DC under the chosen values of the factors is equal to G = 2927 kN.

So if the DC does not rotate, it is rectilinear and its axis is horizontal (β = 0), then the force resisting to its longitudinal motion equals $F_w(0) = \mu \cdot G$ = 585.4 kN. It increases with the increasing of β and a but can be reduced essentially by matching the longitudinal motion with simultaneous rotation.

In Table 1 the calculation results are listed for the value $v = \dot{w}/(\omega d/2) = 1/75$ and different inclination angles β = 0, 5°, $\beta_{fr} = arctg\mu$ = 11.3°, 25°. They represent the values of the longitudinal force F_w, torque M_w and twist angle χ at the end s = 0 and the maximal values of the distributed contact $|f^c|_{max}$ and friction $\left|f_w^{fr}\right|_{max}$ forces found for the operations of the DC lowering and raising. The results demonstrate, that for $\beta > 0$ the initial longitudinal force $F_w(0)$ essentially depends on the operation type. During lowering the friction and gravity forces have opposite directions, so their resultant force is smaller the same force generated in raising. The forces $F_w(0)$, f^c and f_w^{fr} appreciably increase with the angle β enlargement. This effect can be explained by the fact that when β = 0 and the spiral axis is horizontal, the gravity forces do not prestress the DC and the longitudinal force $F_w(s)$ is small too. In this instance the contact force equals only the gravity force. But when the DC is inserted into an inclined bore-hole ($\beta > 0$), the gravity force projection on the DC axis becomes to be essential, the F_w force increases and it compresses more intensively the DC to the curvilinear bore-hole wall. So the contact force grows with the β enlargement which entails growth of the friction force. All the forces and the $M_w(0)$ moment depend also on the direction of the DC rotation (sign before ω).

Some of the attained results for the raising operation at β = 0 and negative ω are represented schematically in Figs. 3–8. Fig. 3 shows longitudinal force F_w as a function of s. It has oscillating character and diminishes to the zero value at $s = S$. It is caused predominantly by the action of the distributed friction force $f_w^{fr}(s)$ (Fig. 4) and gravity force $f_w^{gr}(s)$, which have the same directions in the raising regime. Friction force character is dictated by the resultant $f^c(s) = \sqrt{f_u^c(s)^2 + f_v^c(s)^2}$ of the contact forces $f_u^c(s)$, $f_v^c(s)$. Outline of the $f^c(s)$ force is depicted in Fig. 5. It is significant that all the force functions represent oscillating curves conditioned by the spiral geometry of the DC and slewing the (u, v, w) triad.

Fig. 6 illustrates the variable s dependence of the resultant shear force $F(s) = \sqrt{[F_u(s)]^2 + [F_v(s)]^2}$. It has the smoothed character.

In Fig. 7 is shown the diagram of the s – dependence of the distributed friction moment m_w^{fr}. It causes generation of internal torque M_w varying linearly with small scale harmonics superimposed on it (Fig. 8).

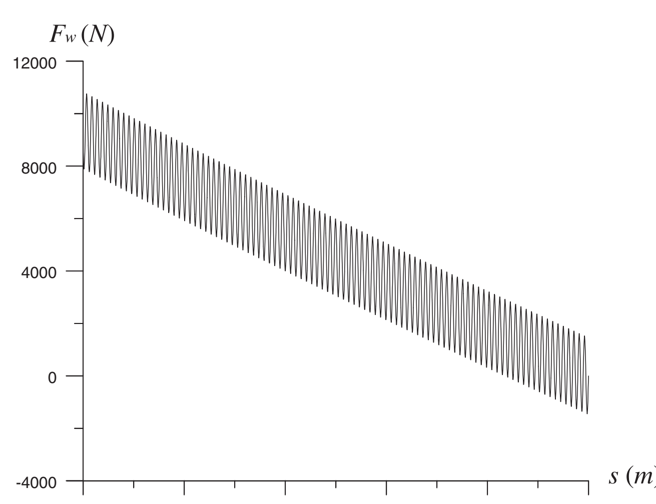

Fig. 3. Internal longitudinal force F_w versus longitudinal coordinate s (spiral imperfections).

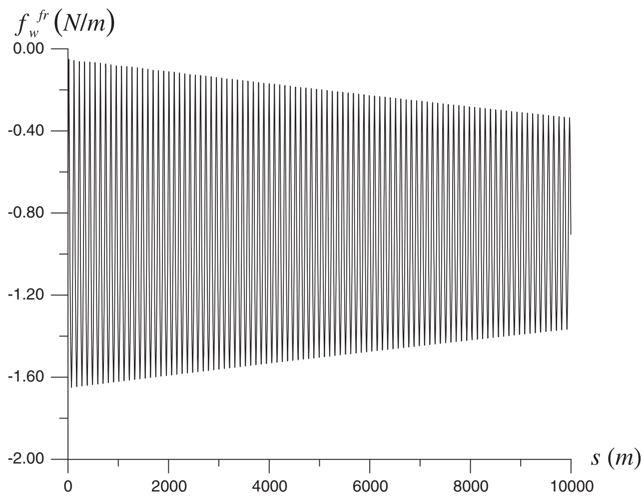

Fig. 4. Distributed longitudinal friction force f_w^{fr} versus longitudinal coordinate s (spiral imperfections).

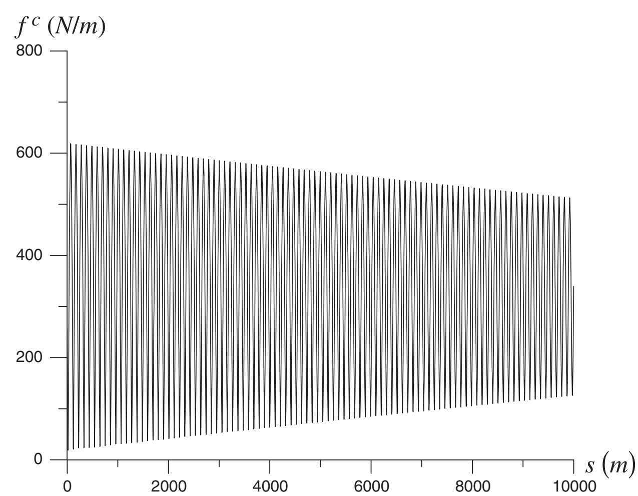

Fig. 5. Resultant distributed contact force f^c versus longitudinal coordinate s (spiral imperfections).

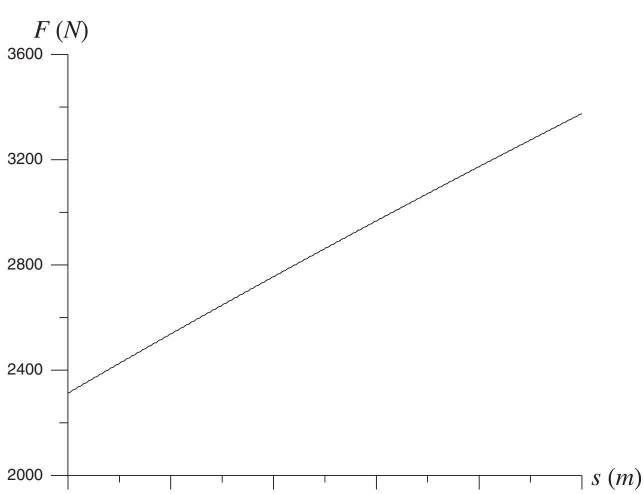

Fig. 6. Internal resultant shear force F versus longitudinal coordinate s (spiral imperfections).

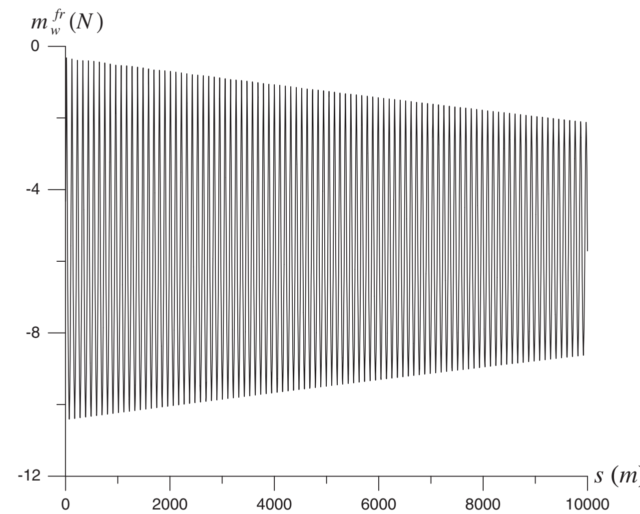

Fig. 7. Distributed friction moment versus longitudinal coordinate s (spiral imperfections).

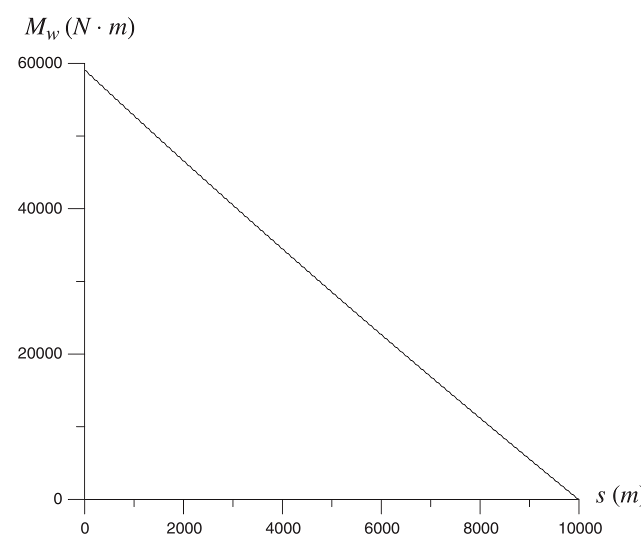

Fig. 8. Elastic torque versus longitudinal coordinate s (spiral imperfections).

The common peculiarity of the built diagrams is that all of them have oscillatory character with small amplitude splashes about linear skeleton lines. The number of the splashes is equal to number 100 of the spiral curve coils.

Analyzing the adduced results one can recognize that all the external distributed forces are oscillating according to the curving of the spiral axis of the bore-hole. But the internal force $F_w(s)$ and torque $M_w(s)$ are integrals of these functions, so they are smoother (Figs. 3, 8).

Further still the resultant bending moment M is at all constant in the spiral bore-hole.

Indeed, according to Eq. (6)

$$M = \sqrt{M_u^2 + M_v^2} = \sqrt{(Ap)^2 + (Bq)^2} = A\sqrt{p^2 + q^2}.$$

In line with Eq. (2)

$$M = A\sqrt{\left(\frac{1}{R}\sin\chi\right)^2 + \left(\frac{1}{R}\cos\chi\right)^2} = A/R$$

and from Eq. (21) it issues

$$R = \frac{a}{\cos^2\alpha} = const, \quad M = \frac{A\cos^2\alpha}{a} = const. \tag{26}$$

The situation is slightly harder for the resultant shear force

$$F = \sqrt{(F_u)^2 + (F_v)^2}.$$

According to Eq. (24)

$$F = \frac{C-A}{RT} + \frac{C}{R}h_2 + 2\frac{A(C-A)}{R}h_2\left(\frac{1}{T} + h_2\right)\left(\sin^2\chi - \cos^2\chi\right). \tag{27}$$

Here the function F is smoother owing to the two first members.

It is also of interest to underline that as the moment M is known, the moments M_u, M_v are not included into constitutive differential equation (23) and the shear forces F_u, F_v can be excluded from this system through the use of their expressions in Eq. (24). So boundary conditions for these variables are not used in system (25).

4. Raising and lowering a drill column in a bore-hole with cosinusoidal imperfections

One of the widespread types of the bore-hole geometry imperfections is also cosinusoidal outline of its axis with different amplitudes a and pitches λ. It is described by the equalities

$$x = \lambda\vartheta\cos\beta/2\pi, \quad y = 0, \quad z = -\lambda\vartheta\sin\beta/2\pi + a\cos\vartheta\cos\beta. \tag{28}$$

Variable ϑ used in these equalities for the cosinusoidal curve parameterization does not coincide with the independent variable s in Eq. (1) which is defined as the length of the bore-hole axial line. For this reason in system (2), (4), (8), (9) the differential ds should be substituted by the value $ds = Dd\vartheta$ (Gulyayev et al., 1992), where

$$D = \sqrt{(dx/d\vartheta)^2 + (dy/d\vartheta)^2 + (dz/d\vartheta)^2} = \sqrt{\lambda^2/(2\pi)^2 + a^2\cdot\sin^2\vartheta}. \tag{29}$$

In this case

$$R = \frac{2\pi}{\lambda a\cos\vartheta}\sqrt{\left(\frac{\lambda^2}{4\pi^2} + a^2\sin^2\vartheta\right)^3}, \quad T = 0 \tag{30}$$

and the system of constitutive Eqs. (23), (24) acquires the form

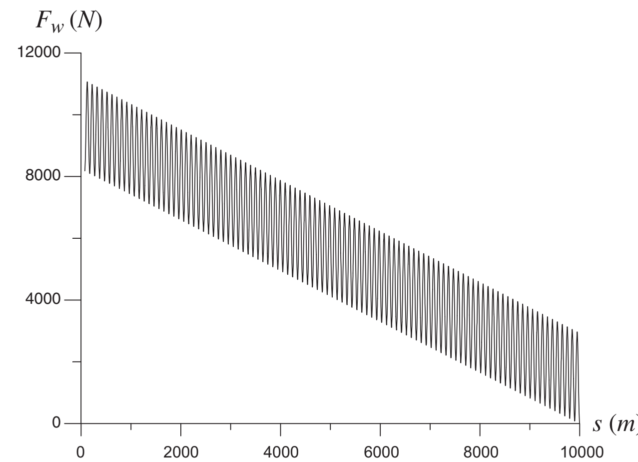

Fig. 9. Internal longitudinal force F_w versus longitudinal coordinate s (cosinusoidal imperfections).

$$\begin{aligned}
&\frac{dh_1}{d\vartheta} = h_2,\\
&\frac{dh_2}{d\vartheta} = \frac{1}{D}\cdot\frac{dD}{d\vartheta}\cdot h_2 - \frac{D^2 m_{fr}}{C},\\
&\frac{dF_w}{d\vartheta} = D\left(\frac{\cos\chi}{R}F_u - \frac{\sin\chi}{R}F_v - f_w^{fr} - f_w^{gr}\right),\\
&F_u = \frac{C}{R}\sin h_1\cdot\frac{h_2}{D},\\
&F_v = \frac{C}{R}\cos h_1\cdot\frac{h_2}{D},\\
&f_u^c = \frac{\sin h_1}{R}m_{fr} + \frac{C}{R^2}\cdot\frac{dR}{d\vartheta}\sin h_1\cdot\frac{h_2}{D^2} - \frac{1}{R}\cos h_1\cdot F_w - f_u^{gr},\\
&f_v^c = \frac{\cos h_1}{R}m_{fr} + \frac{C}{R^2}\cdot\frac{dR}{d\vartheta}\cos h_1\cdot\frac{h_2}{D^2} + \frac{1}{R}\sin h_1\cdot F_w - f_v^{gr},\\
&f_w^{fr} = \pm\mu\left[\sqrt{(f_u^c)^2 + (f_v^c)^2}\cdot\dot{w}/\sqrt{\dot{w}^2 + (\omega\cdot d/2)^2}\right],\\
&m_{fr} = \pm\mu\left[\sqrt{(f_u^c)^2 + (f_v^c)^2}\cdot d^2\cdot\omega/4\sqrt{\dot{w}^2 + (\omega\cdot d/2)^2}\right].
\end{aligned} \tag{31}$$

To analyze the influence of the type of the bore-hole axis imperfections on the drill column bending state at the performance of the

Table 2
The results of calculation of the lowering–raising operations in the cosinusoidal bore-hole of the length S = 10,000 m (a = 5 m, λ = 100 m, v = 1/75).

Operation	β (deg)	ω (sign)	$F_w(0)$ (kN)	M_w (0) (Nm)	χ (rad)	$\lvert f^c\rvert_{max}$ (N/m)	$\lvert f_w^{fr}\rvert_{max}$ (N/m)
Lowering	0	+	−8.181	−51,628	−105	454.2	1.21
		−	−8.181	51,628	105	454.2	1.21
	5	+	216.180	−240,975	−344	4598.2	12.26
		−	216.180	240,975	344	4598.2	12.26
	11.3	+	488.520	−532,771	−745	9945.9	26.52
		−	488.520	532,771	745	9945.9	26.52
	25	+	1055.000	−1,142,110	−1589	21055.0	56.14
		−	1055.000	1,142,110	1589	21055.0	56.14
Raising	0	+	8.181	−51,638	−105	511.2	1.36
		−	8.181	51,638	105	511.2	1.36
	5	+	301.970	−300,729	−407	6275.0	16.73
		−	301.970	300,729	407	6275.0	16.73
	11.3	+	677.930	−662,369	−878	13647.0	36.39
		−	677.930	662,369	878	13647.0	36.39
	25	+	1461.300	−1,418,524	−1869	28993.0	77.31
		−	1461.300	1,418,524	1869	28993.0	77.31

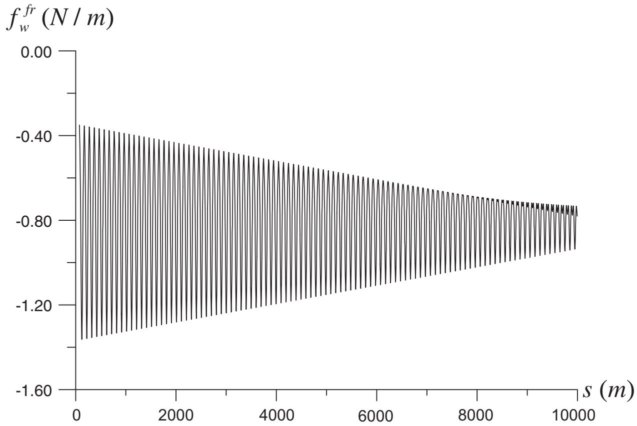

Fig. 10. Distributed longitudinal friction force f_w^{fr} versus longitudinal coordinate s (cosinusoidal imperfections).

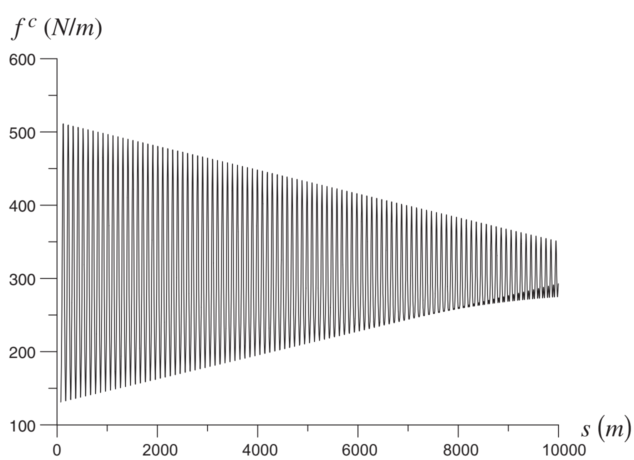

Fig. 11. Resultant distributed contact force f^c versus longitudinal coordinate s (cosinusoidal imperfections).

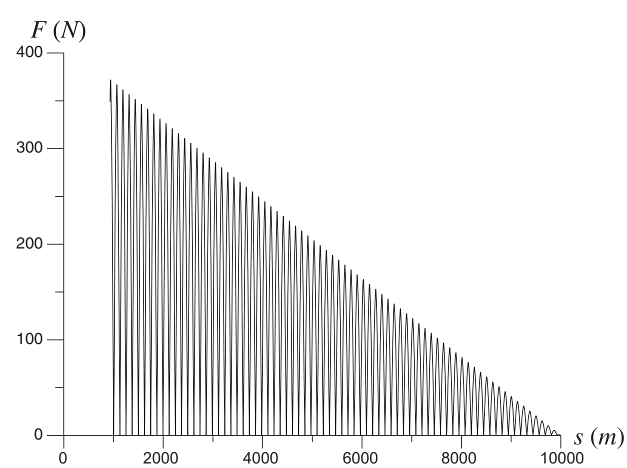

Fig. 12. Internal resultant shear force F versus longitudinal coordinate s (cosinusoidal imperfections)

lowering–raising operations, the cosinusoidal well with the characteristic parameters values used in the foregoing calculations was selected. The total length of the skeleton line makes up $S = 10{,}000$ m,

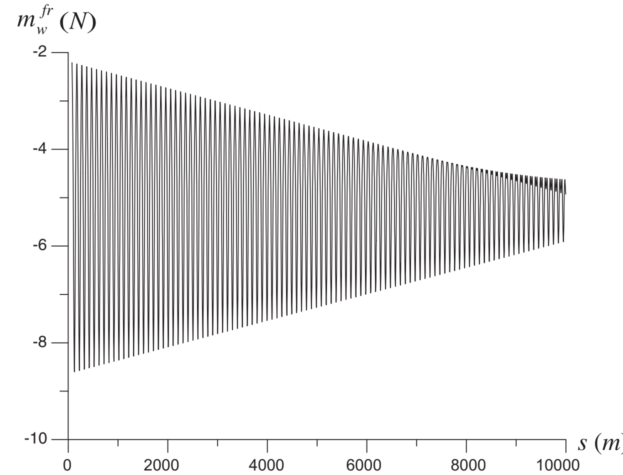

Fig. 13. Distributed friction moment versus longitudinal coordinate s (cosinusoidal imperfections).

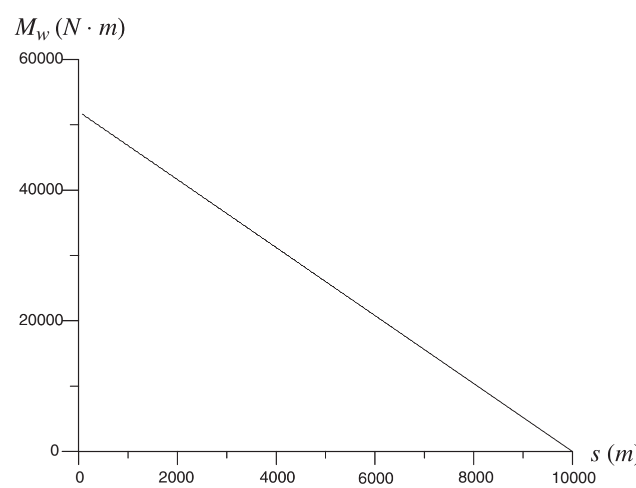

Fig. 14. Elastic torque versus longitudinal coordinate s (cosinusoidal imperfections).

the cosinusoid pitch $\lambda = 100$ m, the amplitude $a = 5$ m. The stiffness and weight characteristics of the DC also remained unaltered.

The calculation results for the case $v = 1/75$ are given in Table 2. They testify, that the general regularities established for the spiral DC are also confirmed for the cosinusoidal one, except that the computational parameters values do not depend on the direction of the rotary velocity ω. The diagrams of the functions $F_w(s)$, $f_w^{fr}(s)$, $f^c(s)$, $F(s)$, $m_w^{fr}(s)$, $M_w(s)$ for the raising regime at $\beta = 0$ are represented in Figs. 9–14, correspondingly. All of them but the moment $M_w(s)$ have oscillating character with conventional period $\lambda = 100$ m.

The curvature radius R (Eq. (30)) is not constant for the cosinusoidal imperfections, as distinguished from the spiral axial line of the bore-hole. In consequence of this the resultant bending moment M is varying function and the resultant shear force F (Fig. 12) also acquires the shape of oscillating curve with maxima at the cosinusoid flex points and zero values at the cosinusoid extremal points.

To correlate influences of the spiral and cosinusoidal imperfections on the resistance forces to the DC movement it is worthwhile to correlate curvature k_{sp} of the spiral curve and maximal curvature k_{cs} of the cosinusoidal one calculated by the formulae

Table 3
Values of the $F_w(0)$ force in spiral and cosinusoidal drill column ($\beta = 25°$).

ν	1/100	1/1	1/100	1/75	1/50	1/25	1/5	1/1
$a(m)$	1	1	5	5	5	5	5	5
$F_w(0)$ (kN) (spiral imperfections)	1284	62,120	1408	1498	1701	2562	181,450	–
$F_w(0)$ (kN) (cosinusoidal imperfections)	1268	12,365	1400	1461	1593	2095	30,784	$2{\cdot}10^9$

$$k_{sp} = \frac{4\pi^2 a}{4\pi^2 a^2 + \lambda^2}, \quad k_{cs} = \frac{4\pi^2 a}{\lambda^2}.$$

For the chosen parameters $a = 5$ m, $\lambda = 100$ m these curvatures have the values k_{sp}=0.01787 m^{-1}, k_{cs}=0.01974 m^{-1}. So the comparison between the values of the dragging force $F_w(0)$ in Tables 1 and 2 indicates that they are larger for the cosinusoidal geometry during the lowering operations and smaller at raising ones.

Included in Table 3 are the results of the $F_w(0)$ force calculation for the operation of the DC raising in a bore-hole with the incline angle $\beta = 25°$, different amplitudes a and ratios ν.

Clearly the longitudinal force has the larger values for the larger amplitudes. It can be also seen that the spiral imperfections are associated with the larger resistance effect, which is more conspicuous for larger ratios ν. At first the dragging force $F_w(0)$ enlarges slowly with the ν enlargement but at $\nu \geqslant 1/5$ the DC seizure happens and the raising operation becomes impossible.

5. Conclusions

The problem about identification of resistance forces acting on a drill column moving in an inclined bore-hole is stated. It is supposed that the well trajectories can have geometrical imperfections in the shape of cylindrical spiral or plane cosinusoidal curves. The system of ordinary differential equations is derived on the basis of the theory of curvilinear flexible elastic rods. With its application the direct and inverse problems of the drill column deforming are formulated for calculation of internal and external resistance forces acting on the drill column tube. Through their use the phenomena of the drill columns motion and their frictional locking inside the bore-holes are simulated for different geometrical imperfections and relations between the velocities and directions of their rotation and axial motion.

The obtained results of computer simulation enable us to draw the following inferences.

1. The resistance friction forces generated due to contact interaction of drill column with a bore-hole wall can be regulated via imposing upon the DC simultaneous axial motion and rotation.
2. The efficiency of this procedure depends on the value of ratio between the velocities of the axial and rotational motions. The ratio reduction leads to essential diminishing the resistance forces while its enlargement is accompanied by their increase. Then at some limit values of this ratio the DC seizure effect happens and the raising procedure becomes impossible.
3. The efficiency of the proposed approach based on the use of the theory of elastic curvilinear rods increases with enlargement of the DC curvature which is typical for bore-holes with short-wave imperfections.

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